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Does the analysis of principal components effectively help in determining actual weights for dimensions of an index? An appraisal in Indian context

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Abstract: *Weights are generally supposed to indicate the relative importance of a dimension variable to explain a particular dependent variable (i.e., a final index), and these weights can be assigned either through individual value judgment, or by using some relevant techniques based upon relative importance of the indices concerned as revealed from the data. Principal Component Analysis (PCA) is playing a major role in determining weights and this principle is clearly based on the degree of variability of the individual dimension-indices. The more the variability, the more will be the assigned weight. Thus, in PCA, the weights are not supposed to be equal and are determined from factor loadings and Eigen-values. In PCA that indicator receives a higher weight which has a higher variance even if that indicator is not directly and even not strongly indicating higher level of importance. In this research agenda, we are to show some major limitations of the Principal Component Analysis and the present study proposes an alternative method to determine actual weights for the underlying dimensions of Human Development Index and in the way has carried out two empirical studieson HDI components of India for the years 1999-2000 and 2007-08.*

Keywords: Index, Principal Component, Average Correlation, Covariance Matrix, Iteration

JEL Classification Codes: C43, O15, O18

Introduction

Development of human beings is considered as a multi-faceted phenomenon and it covers a broad spectrum of economic, social, political, cultural and technical developments. Traditionally development of a nation was understood by the capacity of its economy, whose initial economic condition has been more or less static for a long time, to generate and sustain an annual increase in its GNP or later on, in its per capita GNP. However, as the focus of understanding shifted with the passage of time, development strategies have started emphasizing achievements in the fronts of social indicators like gains in literacy, schooling, health conditions and services etc.

As development involves different aspects of human lives and society as a whole, its measurement is to be done through a composite measurement procedure and this procedure tries to measure average attainment of different ends of human or social lives. Therefore, construction of a composite measurement scale related to development involves three basic instruments: choice of variables, indexing the variables in a suitable manner so that those can be

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mathematically compatible and combining the variables through proper aggregation technique to obtain a certain final comparable value. Broadly the term 'development', means a combination of different parameters with varied characteristics and in narrow sense, if one considers a particular area of development, i.e., either the development of health, or the development of education or the development of living standard (or consumption pattern) of an element, separately these are consisting of different dimensions, different units and obviously with different weights. The measurement and comparability of development indicators (i.e., different dimensions of development) are drawing significant attention for long from the academic circle. As the units of measurement for different indicators are different, and therefore algebraically un-amalgamable, to make them unit freeis an urgent necessity. The method of indexing with a fair number of parameters (i.e., indicators) or broadly with certain dimensions is a popular one with its patronization from the UNDP. To construct this index, along with the actual variate-values, the maximum and minimum values of the same series (or the projected series) are required as either normative goalposts or observed goalposts, as the case may be. The elementary measurement of dispersion, i.e., range is used here to normalize the actual attainment over the minimum attainment of a particular dimension and the index-value can be obtained by using this formula,

Dimension Index = [(Actual Value – Minimum Value) / (Maximum Value – Minimum Value)].

In this case we can have unit-free index-values with [0, 1] as extremes and all values lying inside. In the next phase, the Final Index (FI) is constructed as a linear combination of all the dimension indices, which are not supposed to be perfect substitutes all the time. The selection of appropriate weights for the dimensions (or the underlying parameters therein) is the most crucial part in the construction of any development index and it is discussed elaborately in the following sections of 3, 4 and 5 of this article. Measurement of development indicators along with its comparability among the nations (or states or any part therein) with a new method vis-à-vis with the older one is the most important task ahead and this article intends to put some focus on that direction.

In Section 2 of this article the major objectives of this work are mentioned and in Section 3 a brief survey of available literature on our topic have been presented. In Section 4, a critical appraisal of the Principal Component Analysis as a tool for determination of weights is prepared and probably this section is helping us to identify the research gap. The following sections i.e., the Sections 5 & 6 will be used here to propose a new method of weight determination, namely the 'Iterative Average Correlation Method' (IACM) with some empirical verification in support of it. We are to present two case studies with reference to India's Human Development Index (HDI) and the relative weights of the underlying dimensions therein by applying two different methods, one already exists and the other would be newly proposed. Finally, the Section 7, of this article presents some implications on our proposal and draws several conclusions.

Objectives

In this paper we want to address the following objectives:

a. To present a critical appraisal of the Principal Component Analysis (PCA) with its role in determining weights to individual dimensions of an index.
b. To find out the respective values of Human Development Index for the states of India (for 1999-2000, data published in 2001 and for 2007-2008, data published in 2011), in which

the relative weights of the dimensions are computed on the basis of Principal Component Analysis (PCA).

c. To develop a suitable methodology for constructing a New Human Development Index (NHDI) paradigm for the states of India, and to construct NHDI values for the same couple of periods by using the newly proposed Iterative Average Correlation Method (IACM), for determination of actual weights to dimension indices and to mitigate the encountered problems to a large extent.

A Brief Review of Literature

Literature on theories, methods and analysis of index formation and weight determination for the indicators are vastly available and we have got an opportunity to make a brief survey of most of those works. Since 1990, the United Nations Development Programme (UNDP), with the support from its academicians, have prepared, published and justified the relevant concepts and analytical explanations about data and methodology of the development indicators for constructing HDI of all the nations. We have gone through the methods applied for the purpose and tried to understand the underlying facets of HDI construction. It is an established fact that, determination of actual weights for a formula of weighted average of some parameters is a matter of significant concern for the statisticians and policy-makers. Weights are generally supposed to indicate the relative importance of an explanatory variable amongst others to explain a particular dependent variable. In other words, weights can be clarified as the respective individual explanatory power of a variable to describe an explained variable in the formation of an equation of index.

Weights to indicators can be assigned in a number of ways. One can simply judge the significance of an indicator on the basis of value-judgment and accordingly can assign a weight to it. In technical terms, one can assign equal weights to all indicators or assign different weights to different indicators according to their merit on the basis of acceptable reasoning. Moreover, there are a few available statistical methods like the Principal Component Analysis to determine actual weights for the development related dimensions. We will discuss these methods one after another.

The first one is attaching equal weights to all parameters in explaining a particular dependent variable. The famous UNDP methodology for construction of HDI is based on such principle in which all the dimension indices of Health (H), Education (E) and Living Standard (LS) are given same weight (i.e., 1/3 each), when those are taken into account to form Human Development Index. Even if the UNDP methodology of ‘homogeneous weight principle’ is accepted and applied by the majority of the researchers in the field of Social Sciences, it can seriously be mentioned here that, the UNDP researchers are believed to be prudent enough to select the underlying Dimension Indices (and thereby capability variables) in such a manner that those indicators do carry almost equal weight implying almost equal explanatory capacity for each of them. But this phenomenon of attaching uniform weight cannot be possible in all cases, where the independent indicators are of different nature and are having different degrees of explanatory power in them and thereby, this weighting principle has been criticized as arbitrary. Hopkins (1991) mentioned that there might not be perfect substitutability among the dimension indices and that’s why the concept of attaching equal weights is unjust. Desai (1991) and Ravallion (1997) also opined in favour of flexible weights as equal weights might not reflect the reality.

Secondly, Noorbaksh (1998) and others claimed that the weights to individual indices should also be obtained from the data. Many of them suggested that the coefficients of the first principal component of the individual indices could be used as their weights. Biswas and Caliendo (2002) have also suggested in favour of *Principal Component Analysis (PCA).* Even if PCA has gained significant importance in the arena of weight determination over time, in this paper we are going to present a critical appraisal (in Section 4) of the same (i.e., PCA) and proceed further to propose a new method for the said purpose.

Some Observations on Principal Component Analysis

The theory of Principal Component Analysis (PCA) is accepted by a large number of social scientists as a way out from the complex problem of actual weight determination. The principal components are constructed as a linear combination of the available variables in such a manner that the variance of the linear combination is maximized subject to the constraint that the sum of the squared coefficients must be equal to unity.

It is in fact a dimension-reduction tool that can be used to reduce a large set of variables to a small set that still contains most of the information and moreover, one can transform the concerned correlated variables to a fewer number of uncorrelated variables called principal components by applying this method. Traditionally, the PCA is performed on a square symmetric matrix and the nature of the matrix is of three types – (i) the matrix of the SSCP nature (i.e., pure sums of squares and cross products), (ii) the covariance matrix (i.e., scaled sums of squares and cross products) and (iii) the correlation matrix (i.e., sums of squares and cross products from standardized data).

The analysis of principal components, as mentioned, can be done either through the covariance matrix or through the correlation matrix. Initially the analysis was introduced on the basis of covariance matrix in which the components were determined by variances and co-variances of the variables concerned. Now as the variances and co-variances can be compromised through changes in the units of measurements, the analysis on the basis of correlation matrix was introduced. This analysis on the basis of correlation matrix is same as that on the basis of covariance matrix of the standardized variables. Thus, if the units of measurements of the variables are different, one may be tempted to use the analysis on the basis of correlation matrix. But the problem of using the correlation matrix to run the PCA is that, it provides equal weights to the concerned variables, if the number of independent variables is only two, despite their individual explanatory powers are different. Even for three variables, the result may be the same in the form of securing equal weights for all if the pair-wise correlations of the variables are not different, despite their individual explanatory powers are different. Hence, the usage of correlation matrix as a tool for PCA is not that satisfactory. So far as our present study is concerned, we are dealing with three dimension indices to construct a final index (i.e., HDI), we do not need correlation matrix to run the PCA, as by definition, the dimension indices are unit-free. Hence the customary covariance matrix can be used to run PCA here, as to obtain actual weights for the respective dimension indices. However, the method of PCA, which is based on covariance matrix, put much emphasis on the degree of variability of a particular variable (or, a dimension index) and on the basis of that it determines weight. But in actual sense, variability does not imply true explanatory power of a variable (or an index).

The Principal Component Analysis suggests that if the variances of the dimension indices and the respective co-variances amongst themselves are found almost equal, the weights of those dimensions, obtained through the co-efficient of the first principal component will almost be equal (Noorbaksh, 1998), (Biswas & Caliendo 2002). On the other hand, if the respective variances and pair-wise co-variances of the dimension indices are found unequal, the principal component analysis would supposedly provide unequal weights and the method would probably be considered as more relevant (Biswas & Caliendo 2002). However, the major difficulty of PCA is that it pays much attention to the variability of available data for a particular dimension (indicator) and does not take into account the actual explanatory power of that dimension (indicator). Thus for PCA, more the variability, more would be the assigned weight to a dimension. If X_1, X_2 and X_3 are the three dimension indices then the first principal component can be computed as $(a_1X_1+a_2X_2+a_3X_3)$ such that $V(a_1X_1+a_2X_2+a_3X_3)$ is maximized subject to the normalization condition $a_1^2+a_2^2+a_3^2=1$. Obviously the values of a_1, a_2 and a_3 will be determined by the variances and the co-variances of the respective dimensions. The value of a_1 increases relative to $a_2$and a_3 if variance of X_1increases given the variances of X_2 and X_3 and the co-variances. Now if N is the final index, so that $N = \frac{a_1X_1+a_2X_2+a_3X_3}{a_1+a_2+a_3}$, then the correlation between N and X_1 is given by $r_1 = \frac{COV(N,X_1)}{\sqrt{V(N)V(X_1)}}$. Thus if $V(X_1)$ increases, the coefficient of X_1 in the first principal component would increase. But with this increase in $V(X_1)$, the COV (N, X_1) may increase or remain unchanged or may even fall and if other variances and co-variances remain unchanged, the variability of N explained by X_1, given by $r_1$mayfall and this would reduce the weight of X_1 in the construction of N. The principal component analysis cannot guarantee this result.

The concept of 'rotated component matrix' with a few major components and their Eigen-values sometimes do provide expected result. However, in a particular case if the coefficient of any variable in the first PC comes out as positive and that in the second one as negative, there combined impact would certainly be uncertain and this will create crucial problem for the researchers. One should not try to find an easy way-out by taking the absolute values of all the components to construct rotated component matrix (e.g., The National University for Education, Planning and Administration (NUEPA), under the jurisdiction of Ministry of Human Resource Development, Government of India, did the same in its formation of Educational Development Index (EDI) for the years 2005-06 and onwards, and it is considered as the violation of PCA rule) and obtain the requisite weights because these would be erroneous.

Thus, the search for an improved and better method to assign appropriate weights to underlying indicators of a dimension or of a final index still persists and we will now propose a new method, namely the Iterative Average Correlation Method (IACM) in this context, which will supposedly be helpful in overruling the problems.

The Proposed New Method

As stated earlier, this particular study has tried to offer an alternative measure for estimating human development index and its comprising dimension indices which is based on correlation method. It proposes that the weights of individual dimensions (or indicators) are the proportion of their 'average correlation' values with the final index. This method is likely to attach actual weights to the concerned dimensions and the underlying indicators. This measurement approach of human development index, thus proposed, might be termed as New Approach to Human Development Index (NHDI) and the methodology adopted for obtaining the weights through 'average correlation' values might be named as the Iterative Average Correlation Method (IACM). In accordance with statistical texts, we may say that, 'average correlation' of a particular variable (or dimension) is the average value of its all sorts of correlations, i.e., its simple correlation, its ortho-partial correlation (Mondal, 2008) and its semi ortho-partial correlation(s), if any.

The detailed methodology for understanding average correlation and its significance is given below. Let DI_1, DI_2 and DI_3arethree underlying dimension indices of a composite final index. If it is assumed that the dimension-indices are mutually uncorrelated (i.e., there is no overlapping region among them), their exclusive correlation values with the final index will unambiguously be treated as their true explanatory power and therefore their respective weights. However, if those dimension-indices are mutually interrelated, then their variances [i.e., $V(DI_1)$, $V(DI_2)$ and $V(DI_3)$] and their pair-wise co-variances [i.e., $COV(DI_1, DI_2)$, $COV(DI_1,DI_3)$ and $COV(DI_2,DI_3)$] must have some effective role in determining their respective weights. Among these three dimension indices DI_1 will have higher weight than DI_2, and DI_2 will have higher weight than DI_3 if the correlation between DI_1 and DI_2 is greater than that betweenDI_1 and DI_3, and the correlation between DI_1 and DI_3 is greater than that between DI_2 and DI_3. Larger the difference between these correlations, larger will be the difference of the weights of the dimensions. This weighting principle is based on the assumption that the correlation between any two indices is due to their interdependence and we may not have any specific (and prior) knowledge about the nature of this dependence. Thus, a high degree of correlation between DI_1 and DI_2 is supposed to lead towards higher weights for both of DI_1 and DI_2. To eliminate this problem, simple correlations between the respective dimension indices and the final index cannot be used and the average correlation of them with the final index, as mentioned earlier, can be used to determine their proper weights.

As the final index cannot be calculated unless the weights are determined and as the weights (or the average correlations) cannot be calculated unless the final index is determined, they are to be calculated simultaneously through an iterative process. The process starts with some arbitrarily fixed weights of the individual indices. On the basis of these weights a development index is determined. In the third step average correlations of the individual indices with the development index are obtained and these are used as weights to arrive at the new development index. In the next step we are to have new average correlations and new weights and thereby, another new development index is to be obtained. The process is to be repeated until the values of average correlations do converge to their earlier values and the final weights along with the final development index are to be calculated. All these calculations, in relation to this method proposed, can be obtained only through the application of specific computer programming. We have developed such a programming and on the basis of that, we have performed the empirical analysis given below.

Empirical Analysis with reference to India's HDI over two different periods

We have done a case study with reference to India in support of our methodology and we have used the same data which were earlier used by the Planning Commission of the Government of India to compute HDI for India in two different periods 1999-2000 (published in 2001) and 2007-08 (published in 2011). The detailed calculation and comparison of our findings with the original pones are presented in the following tables Table: 1 and Table: 2.

The major observations are as follows

a. Originally HDI values are calculated on the basis of equal weights to the indicators. We have computed HDI values, on the basis of same data, firstly by the PCA method and then by the IACM method and obtained different results. The ranking of the states have changed as there were changes in the values of their respective HDI. As for example, it can be mentioned that, in 2011, the state of Bihar was placed 21 in equal weights principle, 23 in PCA and again 21 in IACM; whereas the state of Orissa was placed 22 in equal weights principle, 21 in PCA and again 22 in IACM. (Ref: Table 2)

Same type of observations was found in 2001 also. The state of Bihar was placed 19 in equal weights principle, 21 in PCA and again 21 in IACM; whereas the state of Jammu & Kashmir was placed 11 in equal weights principle, 8 in PCA and 9 in IACM. (Ref: Table 1)

b. The values of HDI for the respective states of India have also changed due to a change in the methodology of weight determination. As for example, it is to be mentioned that in 2011, the state of Bihar obtained 0.367 HDI in equal weights principle, 0.333 in PCA and again 0.358 in IACM; whereas the state of Orissa obtained 0.362 in equal weights principle, 0.336 in PCA and again 0.357 in IACM. (Ref: Table 2)

c. It can also be said that there is a possibility of underestimation or overestimation of HDI values (and thereby ranking) for the states if the adopted methodology is not fully appropriate. As for example, in 2011, the position of Bihar was overestimated (0.367) according to equal weights, underestimated (0.333) according to PCA and actual (0.358) according to the proposed IACM. In our view, neither the principle of equal weights nor the method of principal components is totally convincing to determine actual weights to the concerned indicators. Hence, our proposal for IACM is likely to be helpful to solve the problem of underestimation or overestimation of HDI values for the states (or likewise).

Table 1: Computation of HDI (PCA) & NHDI (IACM) vis-à-vis published HDI (1999-2000)

States	Health Index 2000	Education Index 1999-00	Income Index 1999-00	HDI 1999-2000	Rank	HDI (PCA)	Rank	NHDI (IACM)	Rank
Andhra Pradesh	0.521	0.385	0.197	0.368	15	0.332	15	0.351	15
Assam	0.339	0.516	0.152	0.336	17	0.321	16	0.334	16
Bihar	0.506	0.271	0.100	0.292	19	0.247	21	0.270	21
Chhattisgarh	0.341	0.365	0.127	0.278	21	0.257	20	0.270	20
Delhi	0.735	0.816	0.800	0.783	1	0.792	1	0.789	1
Goa	0.363	0.751	0.672	0.595	3	0.634	3	0.619	3
Gujarat	0.562	0.512	0.323	0.466	10	0.441	11	0.455	11
Haryana	0.576	0.512	0.417	0.501	6	0.484	6	0.494	6
Jharkhand	0.434	0.271	0.100	0.268	23	0.232	23	0.251	23
Karnataka	0.567	0.468	0.260	0.432	12	0.399	12	0.417	12
Kerala	0.782	0.789	0.458	0.677	2	0.644	2	0.665	2
Madhya Pradesh	0.363	0.365	0.127	0.285	20	0.261	19	0.276	19
Maharashtra	0.601	0.606	0.297	0.501	6	0.471	7	0.490	7
Orissa	0.376	0.372	0.076	0.275	22	0.245	22	0.263	22
Punjab	0.632	0.542	0.455	0.543	5	0.523	5	0.534	5
Rajasthan	0.520	0.348	0.293	0.387	14	0.361	14	0.373	14
Tamil Nadu	0.586	0.570	0.285	0.480	8	0.450	9	0.469	8
Uttar Pradesh	0.398	0.371	0.179	0.316	18	0.294	18	0.307	18
West Bengal	0.600	0.455	0.210	0.422	13	0.380	13	0.403	13
Himachal Pradesh	0.681	0.636	0.426	0.581	4	0.555	4	0.570	4
Jammu & Kashmir	0.457	0.507	0.431	0.465	11	0.463	8	0.466	9
NE (excluding Assam)	0.567	0.535	0.316	0.473	9	0.447	10	0.462	10
Uttarakhand	0.465	0.371	0.179	0.339	16	0.308	17	0.325	17

Source: India Human Development Report 2001, Govt. of India.

Table 2: Computation of HDI (PCA) & NHDI (IACM) vis-à-vis published HDI (2007-08)

States	Health Index 2007-08	Education Index 2007-08	Income Index 2007-08	HDI 2007-08	Rank	HDI (PCA)	Rank	NHDI (IACM)	Rank
Andhra Pradesh	0.580	0.553	0.287	0.473	15	0.450	15	0.468	15
Assam	0.407	0.636	0.288	0.444	16	0.430	16	0.441	16
Bihar	0.563	0.409	0.127	0.367	21	0.333	23	0.358	21
Chhattisgarh	0.417	0.526	0.133	0.358	23	0.333	22	0.353	23
Delhi	0.763	0.809	0.678	0.750	2	0.742	2	0.748	2
Goa	0.650	0.758	0.443	0.617	4	0.598	4	0.613	4
Gujarat	0.633	0.577	0.371	0.527	11	0.506	11	0.522	11
Haryana	0.627	0.622	0.408	0.552	9	0.534	9	0.548	9
Jharkhand	0.500	0.485	0.142	0.376	19	0.346	20	0.369	20
Karnataka	0.627	0.605	0.326	0.519	12	0.495	12	0.514	12
Kerala	0.817	0.924	0.629	0.790	1	0.773	1	0.786	1
Madhya Pradesh	0.430	0.522	0.173	0.375	20	0.352	19	0.370	19
Maharashtra	0.650	0.715	0.351	0.572	7	0.546	7	0.566	7
Orissa	0.450	0.499	0.139	0.362	22	0.336	21	0.357	22
Punjab	0.667	0.654	0.495	0.605	5	0.591	5	0.602	5
Rajasthan	0.587	0.462	0.253	0.434	17	0.409	17	0.428	17
Tamil Nadu	0.637	0.719	0.355	0.570	8	0.546	8	0.565	8
Uttar Pradesh	0.473	0.492	0.175	0.380	18	0.355	18	0.374	18
West Bengal	0.650	0.575	0.252	0.492	13	0.461	14	0.485	14
Himachal Pradesh	0.717	0.747	0.491	0.652	3	0.633	3	0.647	3
Jammu & Kashmir	0.530	0.597	0.459	0.529	10	0.522	10	0.527	10
NE (excluding Assam)	0.663	0.670	0.386	0.573	6	0.550	6	0.568	6
Uttarakhand	0.530	0.638	0.302	0.490	14	0.469	13	0.486	13

Source: India Human Development Report 2011, Govt. of India.

Conclusion

Human Development is reflected in attainment of different ends of human lives. Creation of capacities is a pre-requisite for such attainment and for some ends attainment is not directly measureable. Translation of capacities to achievements depends on whether these capacities are accessible to human beings. Construction of different dimension indices along with the HDI helps us in obtaining a complete picture of human development. To know about the relative importance of different indicators of development was not the priority of our social planners for long and later on, the analysis of principal components paved the way for objective determination of relative importance through weights. The present article shows that correlations observed among the variables help us in many ways to count their relative importance, and correlation measures the degree of linear association between two or more variables. This article has succeeded in establishing the fact that relative importance of different variables in the construction of HDI is different.

The modified methodology applied by us helps us eliminating the most crucial allegation against the HDI that it attaches equal weights to all three dimensions of human development. For developing countries the relative importance of dimension of income should be greater than those for the developed countries and this is exactly what we have achieved in our methodology. Finally, we can conclude that, attaching weights to respective dimensions of a final index is a matter which is not supposed to be looked down upon. And if weights are to be chosen, neither we should follow the equal weight principle through subjective value judgment nor we should follow the Principal Component Analysis. Rather we should pay much attention to our proposed method of Iterative Average Correlation of attaching weights to the development dimensions which we think will be more likely to show the actual explanatory powers of the concerned indicators.

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